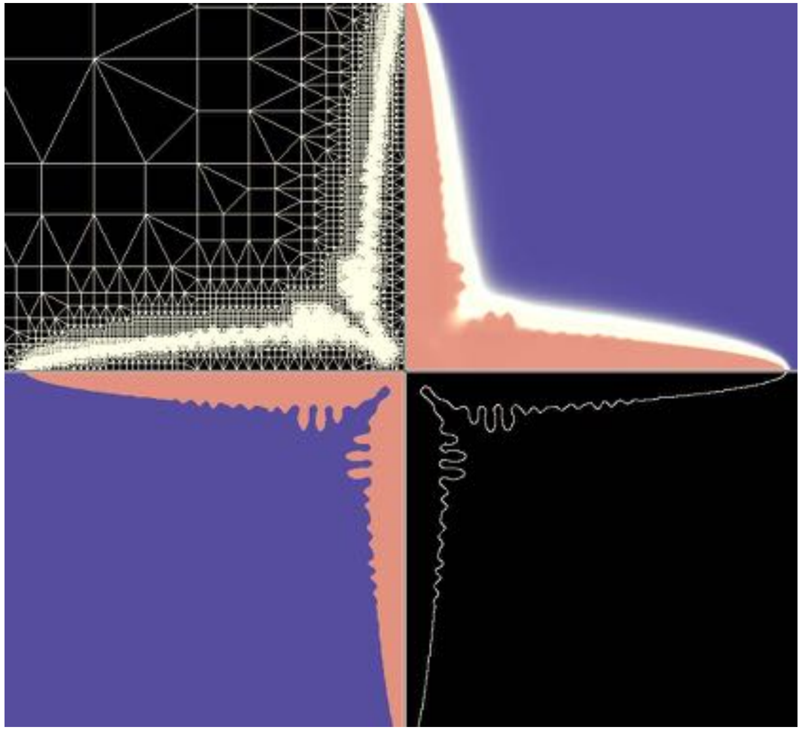


Introduction to Phase-field Modeling in Computational Mechanics

Christian Peco

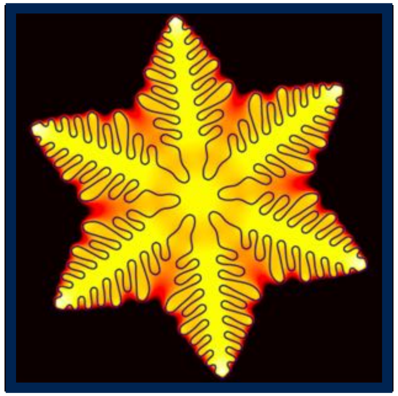
Penn State University - Teaching Seminar

Motivation : problems with interfaces

Two phases can be identified

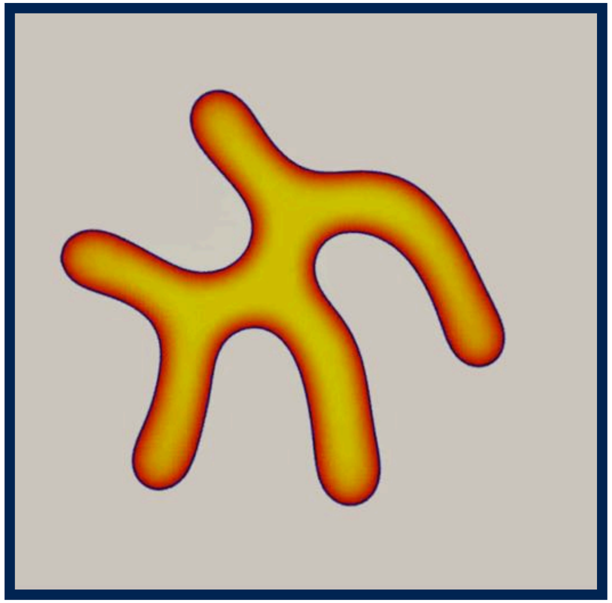
Interface is complex and evolves

Solidification

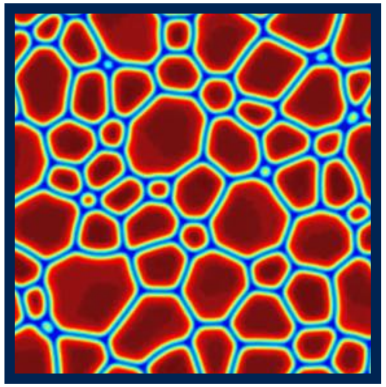


Fracture

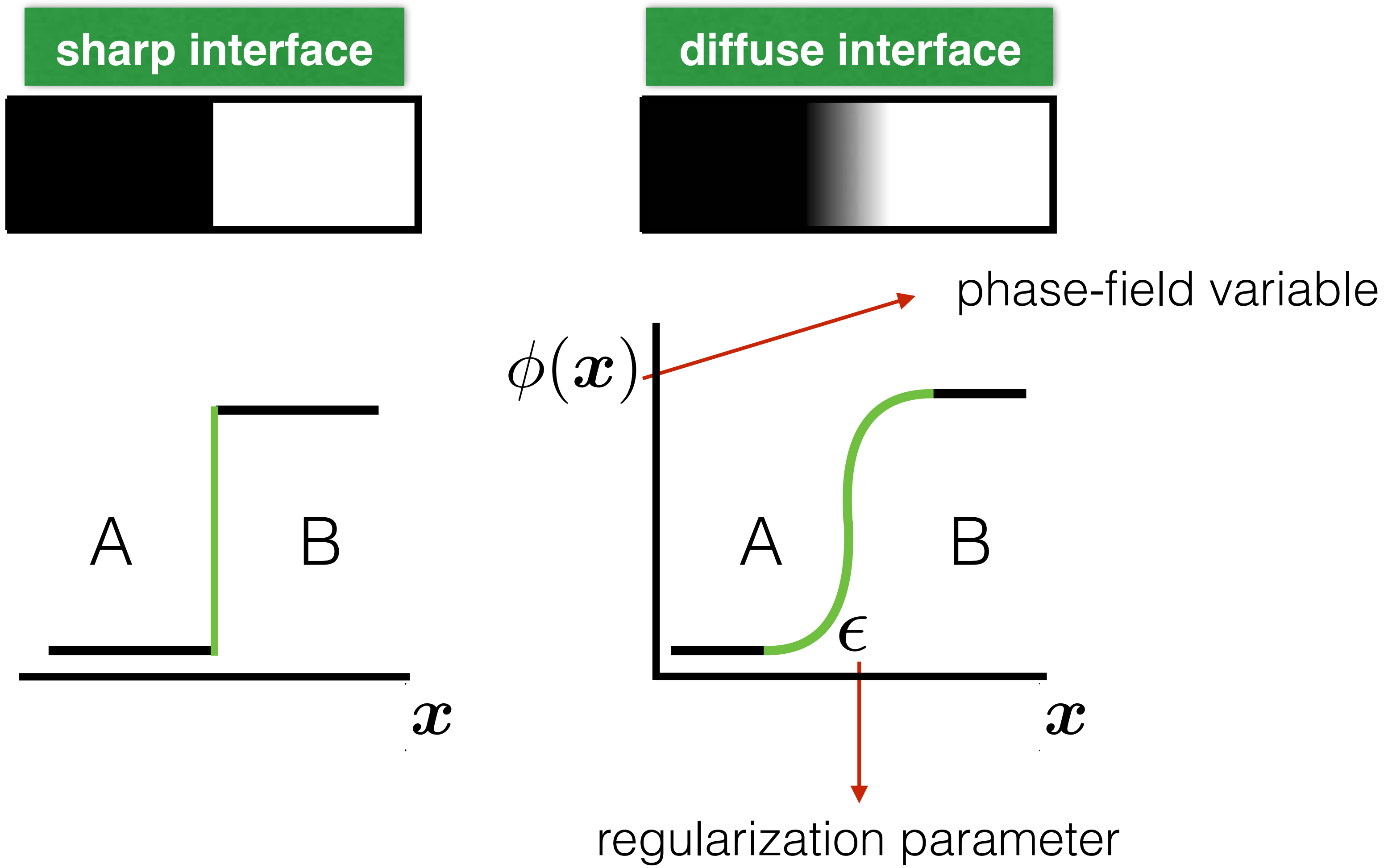
Membranes



Microstructure



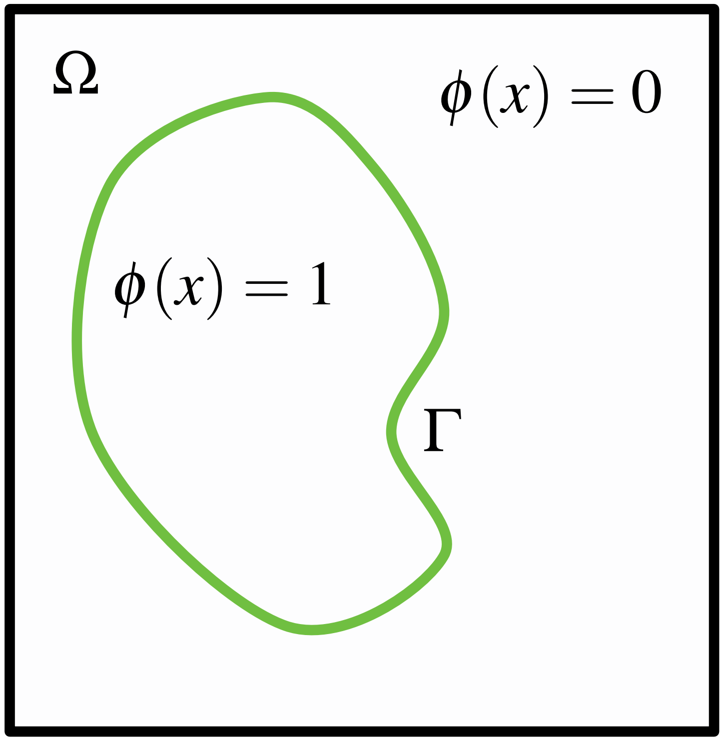
Phase-field Modeling Basics



Continuous on the domain
Avoid interface tracking

→ **Can we rewrite our sharp interface model in terms of the phase-field?**

Phase-field and Geometry



Volume

$$\int_\Omega \phi \, d\Omega$$

Area

$$\int_\Omega \left[\varepsilon |\nabla \phi|^2 + \frac{1}{\varepsilon} F(\phi) \right] \, d\Omega$$

Curvature

$$\varepsilon \Delta \phi + \frac{1}{\varepsilon} F(\phi)$$

Brittle Fracture: Sharp Interface Approach

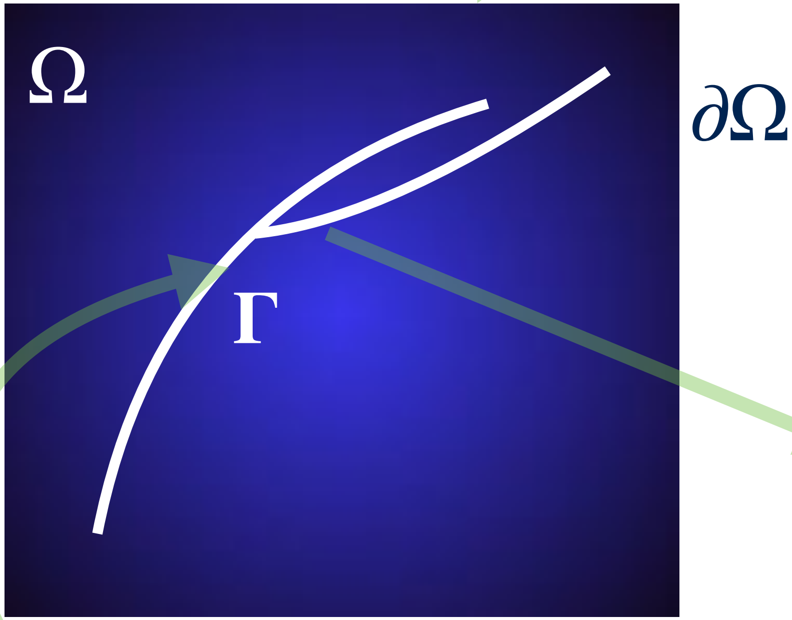
Linear Elastic Solid

$$W = E(u) : \mathbb{C} : E(u)$$

Boundary conditions

$$u = u^* \quad on \quad \partial\Omega_u$$

$$\sigma \cdot n = t^* \quad on \quad \partial\Omega_t$$



Interface behavior

$$f(G_c)$$

Boundary condition

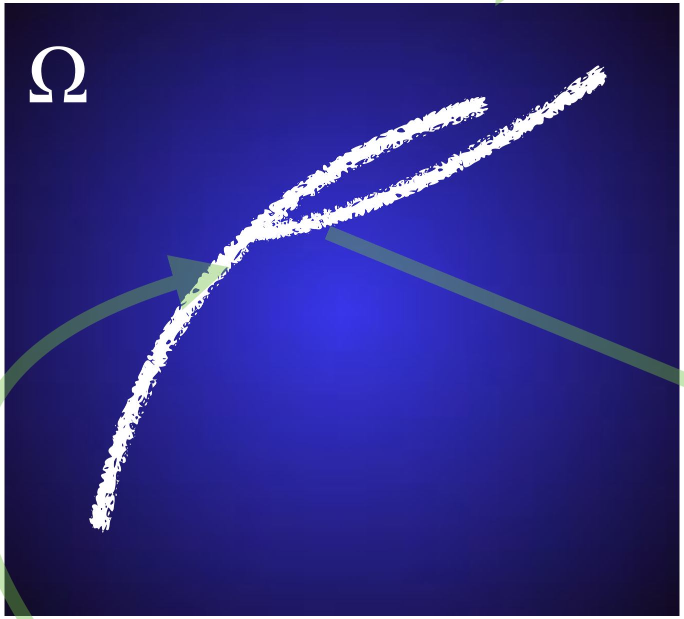
$$\sigma \cdot n = 0 \quad on \quad \Gamma$$

Brittle Fracture : Phase-field Model Derivation

Linear Elastic Solid

$$W = (1-\phi)^2 E(u) : \mathbb{C} : E(u)$$

fracture
$\phi(x) = 1$
undamaged
$\phi(x) = 0$



$\partial\Omega$

$\partial\Omega$

Boundary conditions

$$u = u^* \quad on \quad \partial\Omega_u$$

$$\sigma \cdot n = t^* \quad on \quad \partial\Omega_t$$

~~Interface behavior~~

Phase-field PDE

~~Boundary condition~~

$$\cancel{\sigma \cdot n = 0 \quad on \quad \Gamma}$$

Brittle Fracture : Phase-field Model Derivation

$$\Psi(u,\phi) = \underbrace{\boxed{(1-\phi)^2 E(u) : \mathbb{C} : E(u)}}_{\text{elastic energy}} + \underbrace{\boxed{G_c \left(\frac{\phi^2}{2\varepsilon} + \frac{\varepsilon}{2} |\nabla \phi|^2 \right)}}_{\text{fracture energy}}$$

fractured area

Evolution Equations

$$\nabla \cdot \left(\frac{\partial \Psi(E,\phi)}{\partial E} \right) = 0$$

$$\frac{\partial \phi}{\partial t} + M \left(\frac{\partial \Psi(E,\phi)}{\partial \phi} + \nabla \cdot \frac{\partial \Psi(E,\phi)}{\partial \nabla \phi} \right) = 0$$

Pros vs Cons

Avoid explicit tracking of interface ✓

Can handle complex geometries and topology changes ✓

The phase-field encodes the physics ✓

Dimension increase

Sharp gradients

<u>Computational cost</u> ✗

Parameter choice ✗

Take Home Message

Consider phase-field models when:

1. Evolving complex interfaces
2. Multiphysics, different energy contributions

Remember:

1. The regularization parameter is important
2. Phase-field comes with computational cost
   (most of times it will pay off)